Preliminary Mathematical Analysis of a Rigid-Airfoil, Hydrofoil Water Conveyance

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An optimum configuration for a rigid-airfoil, hydrofoil water conveyance of the sailcraft type is described. The historical background of such craft is discussed, and the various developments leading to the current state-of-the-art are reviewed. The basic equations of motion for an optimized craft are developed and analyzed in order to indicate the efficiency of this novel approach to one of man's most ancient means of transportation.

Introduction

THIS paper describes an optimum configuration for a rigid-airfoil, hydrofoil water conveyance of the sailcraft type and discusses a preliminary mathematical analysis of its steady-state behavior over an ideally smooth water surface. Before the configuration is presented, it seems appropriate to review briefly the various leads in sailcraft technology that were followed in order to develop, what is believed to be, the optimum configuration. It is also noteworthy to recognize that hydrofoil-sailcraft technology represents a practical wedding of the aerodynamic and hydrodynamic sciences. It even exhibits serendipitous relations with aerospace science; in fact, one of the pioneers in the field of hydrofoil sailcraft is Robert R. Gilruth, Director of the Manned Spacecraft Center of the National Aeronautics and Space Administration.

The great speed of light-displacement sailcraft has been recognized for many years. The key to their performance is that they are relatively small, and the lateral stability to carry considerable sail (0.06 m^2/kg) can be provided by moving the crew to the windward rail, thereby shifting the center of mass. Catamarans are reported to have sailed over 25 mph. The key here is to counteract heeling moments by maintaining the line of the restoring (buoyant) force between the two hulls; thus again providing very high lateral stability, at least until the windward hull is lifted clear of the water. Aside from progress towards large sail-area/mass quotients for sailcraft (e.g., from 0.02 m^2/kg for yachts to 0.06 m^2/kg for catamarans), and improved methods for counteracting heeling forces and torques, two other rather novel concepts have been added to sailcraft technology. The first of these is the rigid airfoil developed in 1949 by William P. Carl, Jr.[1] More recent developments in this area of wind propulsion are as follows: a) the system of multiple, plastic airfoils used for propelling a catamaran, on exhibition in 1965 at Lucerne, Switzerland; b) coupling of airscrews to waterscrews as suggested by A. B. Bauer[2] and, in connection with frictionless sailcraft devices, by A. G. Hammitt;[3] and c) the variable-camber, biplane rigid-airfoil system under development by E. Morris Wright. The second novel approach has been to make use of hydrofoils instead of displacement hulls for sailcraft. Robert R. Gilruth patented such a craft in 1955.[4] J. G. Baker, in 1956, designed and sailed such a craft with the assistance of Professor Robert Cannon.[5] The sailcraft was demonstrated to move at speeds up to 35 mph. Bernard Smith in his book, *The 40-Knot Sailboat*,[6] and in his paper, "The Aerohydrofoil,"[7] describes what he terms the "Aerohydrofoil" in which the concept of a hydrofoil system voluminous enough to displace the craft's weight is investigated.

In light of the multiplicity of development in sailcraft technology, it seems appropriate to identify exactly what goals are being sought by the advanced sailcraft designers. Let us consider a simple model of sailcraft forces. Sailcraft propulsion is rather unique in that, except on a run, the thrust does not lie entirely opposite to the drag force vector of the hull. This raises two problems: first, one seeks to maximize the component of sail thrust opposite to drag; and second, one requires both a side thrust to counteract the component of sail thrust normal to the craft's velocity and a couple to counter the aerodynamic-thrust couple occasioned by the center of effort of the sail(s) being above the center of mass of the craft and the center of effort of the hull(s) being below. In conventional sailing yachts the hydrodynamic lift of the hull form (or of a center board) yields the required counteracting normal thrust while a displacement of the center of buoyancy and/or a shift of the center of mass of the craft (by the crew shifting their weight to windward) creates a balancing couple. The speed of the sailcraft is clearly dictated by the equilibrium established between hydrodynamic drag and the component of aerodynamic thrust diametrically opposed to this drag. Thus, one goal is to decrease hydrodynamic drag and the other goal is to increase the aerodynamic thrust component. The situation is not quite as simple as this since it is often not possible to sail on the preferred course, e.g., one cannot (usually) sail directly into the wind‡; and the speed made good to windward, when the sailcraft is moving at an angle to the wind, becomes important. In any event, it is

Received July 1, 1970; revision received June 24, 1971. Presented at the Second AIAA Symposium on the Aer/Hydronautics of Sailing, The Ancient Interface II, April 18, 1970, Costa Mesa, Calif., *AIAA Monograph*, Vol. 9, edited by A. G. Hanimit, Western Periodicals Company, North, Hollywood, Calif., 1970, pp. 131–156. The research for this paper was supported by Transportation Sciences Corporation; the manuscript was prepared with the consent of the McDonnell Douglas Astronautics Company (MDAC). Valuable assistance was provided by J. J. Mangham and M. B. Rein, Technical Documentation and Graphic Services Department, Logistics Support, MDAC.

Index categories: Surface Vessel Systems, Vessel and Control Surface Hydrodynamics; Marine Vessel Design (Including Loads).

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‡ Unless airscrews are connected to waterscrews and the craft has an initial windward velocity; see Refs. 2 and 3.

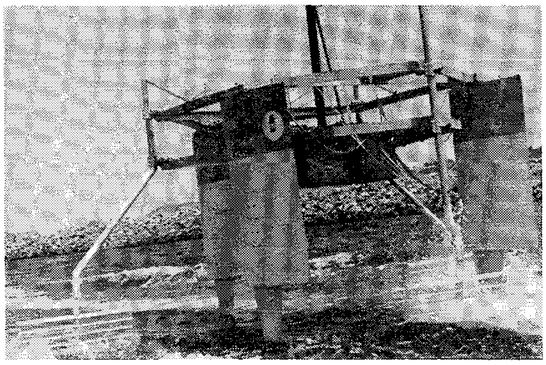

Fig. 1 Aft view of prototype craft.

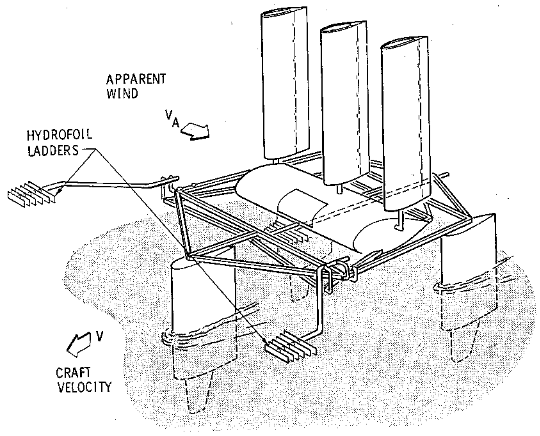

Fig. 2 Three-airfoil craft with trimaran hull form, float-borne with hydrofoils retracted.

clear that better hull forms having lower hydrodynamic drag, and better sails having more unit area per unit sailcraft mass and/or more aerodynamic thrust per unit area, are desirable goals. Increasing sail area, as is accomplished in light-displacement hulls, improves performance, as does the use of rigid airfoils which exhibit higher thrust per unit area than do conventional cloth sails. An added advantage of the rigid airfoils is that their higher lift-to-drag ratio, relative to cloth sails, permits one to point closer into the wind (this will be demonstrated in a subsequent section). The employment of hydrofoils has two purposes: first, to reduce the hydrodynamic drag; and second (if the hydrofoils are deployed optimally) to counteract more efficiently the normal component of aerodynamic thrust and overturning couple. The former result is achieved by lifting the displacement hull (or, in the case of Smith's aerohydrofoil, a portion of the hydrofoil) clear of the water, thereby, reducing drag. The reduction in hull drag is especially dramatic at high speeds where a displacement hull would suffer from large wave drag, which effectively sets an upper limit to a sailcraft's speed. The advantage in reducing hydrodynamic drag does not always lie with the hydrofoils, however, since at low speeds through the water (below the optimum lift-off speed) they may represent appendages having unwanted drag, i.e., drag in excess of that of the unsupported hull. This is true even though a portion of the hull is elevated above the water prior to complete suspension of the craft on the hydrofoils, i.e., prior to liftoff.[8]

Closely related to the force equilibrium concept is that of torque equilibrium. As already noted, on conventional yachts the overturning couple produced by the athwartship sail thrust component (normal to the yacht's velocity) is counteracted by a buoyancy-point center-of-gravity couple. Thus, one finds that conventional racing yachts have heavy lead keels to counteract this heeling couple. The use of multiple hulls also serves this purpose, and, since little or no dead weight is required, offers advantages over lead-keel single-hulled boats. In general, the heeling of a craft required to null out the roll couple has two detrimental effects. First, in the heeled-over position, hull drag is materially increased, accompanied by a resulting loss in sailcraft speed. Second, since the sail is no longer contained in an approximately vertical plane, there is a loss in aerodynamic efficiency. As well as reducing roll it is also advantageous to reduce yaw. Because the sail's aerodynamic thrust force and the hull's hydrodynamic force do not pass through the center of mass (when observed in the plan view), yaw torques exist which are nulled out at the expense of sailing the yacht in cocked fashion relative to its true velocity. One finds the hull moving essentially at an angle-of-attack relative to the water, which is, of course, essential in order to produce the athwartship hull lift or normal force which was mentioned previously. Such an angle, which is termed leeway, also materially increases hull drag. The remaining torque is pitch, which affects the craft's optimum entry into waves and its seaworthiness.

Based upon the foregoing, admittedly sketchy, analysis one can identify four primary goals in sailcraft design: 1) maximization of aerodynamic thrust per unit sailcraft mass (as accomplished by a large sail-area to sailcraft-mass quotient and/or a large lift coefficient for rigid airfoils); 2) minimization of hydrodynamic drag (as accomplished by low-drag hull forms and/or hydrofoil systems); 3) minimization of heeling (as accomplished by counteracting the airfoil thrust component normal to the craft's velocity without producing a large heeling (roll) couple); and 4) minimization of leeway (as accomplished by reducing or nulling out all yaw torques).

Of course, a myriad of other factors exist which affect optimum sailing, such as maneuverability, acceleration, vulnerability to wind gusts, general stability, sea-worthiness (in itself involving numerous subfactors), crew comfort, aesthetic form, public acceptability, safety, reliability, maintainability, challenge for the sportsman, etc. In what follows we shall, however, restrict ourselves entirely to the optimum design based upon fundamental force and torque laws; and for simplicity of analysis, we shall consider operation of the craft only on an ideally smooth water surface.

Optimum Configuration

The four goals of sailcraft design noted in the foregoing section can all be achieved through the use of a combination rigid-airfoil and hydrofoil configuration. It is assumed to be self-evident that rigid airfoils may well outperform and could be the successors to conventional cloth sails, especially in view of recent breakthroughs in the construction of lightweight structures for aerospace applications. A 16-ft, full-scale prototype test vehicle has been fabricated. Figure 1 shows the craft underway with steerage locked in place.

Figure 2 is a sketch of the three-airfoil craft exhibiting a trimaran hull form (i.e., having three buoyancy chambers or floats) with the hydrofoils stowed in a retracted position prior to the craft achieving the optimum lift-off speed.

Figure 3 is a photograph of the full-scale prototype (without the rigid airfoils) with the hydrofoils retracted.

The hydrofoils, except at very low speeds, now outperform conventional light-craft displacement hulls (and have for

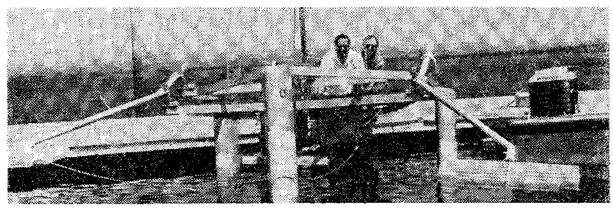

Fig. 3 Prototype shown with hydrofoils retracted.

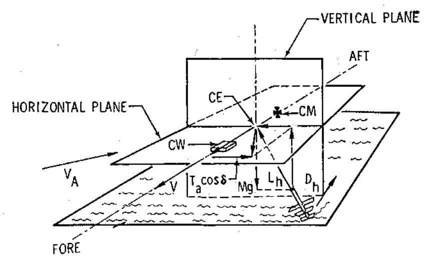

Fig. 4 Optimum configuration schematic.

over seventy years).[9] (We recognize this, but realize that current problems exist in hydrofoil cavitation, optimum control, etc.) There will always be advantages to large airfoil-area/sailcraft-mass quotients, and this concept is subject to continual improvement as new structural materials and construction techniques are developed. The removal of roll, pitch, and yaw torques can be accomplished by an appropriate arrangement of the hydrofoil center of effort, the airfoil's aerodynamic center of effort, and the craft's center of mass. Since this arrangement may not be obvious and, furthermore, since it is the very essence of the sailcraft configuration to be discussed, it is described in detail in the following paragraphs.

The basic objectives of the optimum configuration, as indicated in Figs. 4, 5, and 6, are as follows: a) to maintain the hydrofoil lift component, L_h, (resultant force vector of a hydrofoil system as projected on a plane normal to the craft's velocity vector), inclined to leeward at the proper strut angle, γ, and intersecting the craft's fore-to-aft line of symmetry so as to satisfy the force-equilibrium diagram shown in Fig. 5, and thereby eliminate roll (heeling); b) to place the force vector of the rigid airfoil array $\mathbf{T}_a$ in a horizontal plane that contains this line of symmetry; and c) to maintain the center of mass, CM, also on this line of symmetry. The yaw of the craft produced by the horizontal component of the resultant force vector of the hydrofoil system, i.e., drag component, D_h acting below and leeward of the CM, is controlled and nulled out by moving the center of effort, CE, of the propulsive system, e.g., the mast of the rigid airfoil(s), forward or aft on the craft along the line of symmetry. Thus, an equal and opposite moment is produced to null out yaw. A counterweight, CW, (e.g., crew and/or cabin) is shifted along the line of symmetry in order to maintain the craft's center of mass at a location that will null out the pitch.

The hydrofoils may be of several forms, e.g., variable-area or ladder configuration and sub-cavitating, fully wetted, base ventilated, super-cavitating, etc., in operation. They may be straight, or, preferably, swept back.[10] For the sake of a specific example, we will deal with a ladder configuration, subcavitating, hydrofoil system exhibited schematically by the hydrofoil-borne craft in Fig. 7 and photographically by the full-scale prototype (shown during tow testing) in Figs. 1 and 8. Size and separation at the hydrofoils are compromises between a large lift-to-drag ratio, structural integrity,[11] and efficiency under various operating conditions. A mathematical algorithm for establishing the optimum compromise was developed by Baker and programmed in BASIC computer language at the University of California at Los Angeles by Douglas,[12] which includes a variation of hydrofoil angle-of-attack up the ladder.

Mathematical Analyses

The hydrodynamic drag of a hydrofoil-suspended water conveyance is given by

$$D_h = Mg/(L/D)_h \tag{1}$$

where D_h is the drag force (Newtons), M is the craft's mass (kg), g is the acceleration of gravity (9.8 m/sec²), and $(L/D)_h$ is the lift-to-drag ratio of the hydrofoils.

The calculation of the lift-to-drag ratio is not a simple matter since it depends upon the type of hydrofoil (e.g., subcavitating, supercavitating, etc.), other submerged structure (e.g., struts, propellers, etc.),[13] the partial submergence of hydrofoils whose spans are inclined to the water surface, water surface conditions, etc. In general, $(L/D)_h$ decreases with the craft speed.[14] A simple linearized approximation is

$$(L/D)_h = (L/D)_{h0} - (L/D)_{h1}V \tag{2a}$$

where the coefficient, $(L/D)_{h1}$ has dimensions of sec/m and V is the speed of the craft relative to the water in m/sec. From Altmann's calculations,[14] for example, $(L/D)_{h0} = 38$ and $(L/D)_{h1} = 0.83$ sec/m in the case of subcavitating hydrofoils of the surface-piercing variety; whereas Johnson and Tulin[15] find $(L/D)_{h1} = 25$ and $(L/D)_{h1} = 0.515$ sec/m. It is often more convenient to rephrase Eq. (2a) by the following:

$$(L/D)_h = (L/D)_{h0} - (L/D)_{h1}(V - 2.75) \tag{2b}$$

where, e.g., $(L/D)_{h0} = 17.0$ and $(L/D)_{h1} = 0.476$ sec/m.

The initial tow tests of a full-scale prototype indicated an $(L/D)_h$ of 10 or more for speeds of about 2.5 m/sec and a smaller value for $(L/D)_{h0}$. In this test, however, some parasitic drag caused by the forward float may have interfered with a true estimate of hydrofoil-borne $(L/D)_h$. This parasitic drag was due to the downwardly inclined tow rope, which forced the forward float to pitch periodically into the water. This problem will be avoided in future tests by modifying the towing arrangement and installing, on all the floats, glove tanks whose shapes and locations are selected to yield minimum wave drag at the Froude number at optimum lift-off speed. Furthermore, in the initial tow tests, the hydrofoils were set at too large an angle of attack, which did not serve to maximize the lift-to-drag ratio. The rather large numerical value of the $(L/D)_{h0}$ coefficient implies lift-to-drag ratios on the order of 8 or more for most operational craft speeds. Such large values of $(L/D)_h$ may seem surprisingly high to those who have experimented with hydrofoil-borne water conveyances. The larger $(L/D)_h$ is a direct result of the pivoted hydrofoil concept of the design. In most other hy-

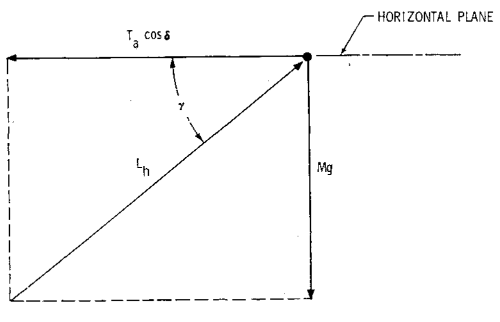

Fig. 5 Equilibrium of forces in vertical plane, normal to the craft's velocity vector, (leeward to left).

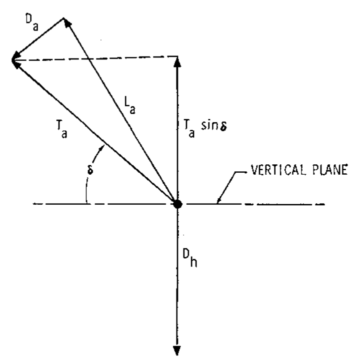

Fig. 6 Equilibrium of forces in horizontal plane, that contains fore-aft line of symmetry.

drofoil craft, especially of the sailboat variety, V- or U-shaped hydrofoils are employed to provide stabilizing forces acting inwardly from opposite sides toward the longitudinal, fore-to-aft, line of symmetry of the craft. Unfortunately, hydrofoils of this type suffer the undesirable characteristic of creating opposed inwardly directed horizontal forces acting inefficiently against each other and resulting in increased induced drag. Thus, such conventional craft will exhibit degraded lift-to-drag ratios when compared to craft exhibiting the pivoted strut design.

The hydrofoil lift-to-drag ratio, $(L/D)_h$, will vary with craft speed, V, strut angle, γ (Figs. 7 and 8), height of the seas, H_1, and direction of the true wind, β_T. The latter two effects account for ocean roughness. The variation of $(L/D)_h$ with V is better approximated by the second-order polynomial:

$$(L/D)_h = (L/D)_{h0} - (L/D)_{h1}V - (L/D)_{h2}V^2 \qquad (3)$$

than by the simpler Eqs. (2a) and (2b). Rather realistic numerical values for these coefficients are: $(L/D)_{h0} = 17.0$, $(L/D)_{h1} = 0.475$ sec/m, and $(L/D)_{h2} = 3.46 \times 10^{-3}$ sec²/m².

With respect to the strut angle, γ, it seems reasonable to expect that as the strut angles up, and γ becomes less than 90° (the vertical strut angle) there will be some hydrofoil-tip surface breaking. Such an effect is, in fact, computed in the program developed by Douglas.[12] Roughly, one can account for γ by multiplying Eq. (3) by

$$(1 - X_{27}|\cos\gamma|) \qquad (4)$$

where X_{27} will be a fitted coefficient, which tentatively has to be taken as 0.20. The height of the seas, H_1, is taken as 0.3 m for $V_T < 4$ m/sec. For wind speed in excess of 4 m/sec, we take the height of the seas to be

$$H_1 = (0.473)V_T - 1.425 \quad \text{m} \qquad (5)$$

The variation of $(L/D)_h$ with H_1 also depends upon the angle between the craft's velocity vector, $\mathbf{V}$, and the velocity of the seas, $\mathbf{V_w}$, which is $\beta_T + \mu$ (ordinarily, we'll assume μ to be zero). For example, when $\beta_T + \mu$ is near zero (heading into the weather), one would expect $(L/D)_h$ to be more degraded than when $\beta_T + \mu$ is near 180° (a following sea). Thus, we can employ the following adaptation of Eq. (4) of Marks et al.[16] as another factor of $(L/D)_h$:

$$1 - X_{24}(H_1 - 0.3)[X_{25} - X_{26}\sin^2(\beta_T + \mu)] \qquad (6)$$

If we arbitrarily determine that for $\beta_T + \mu = 0$ and $H_1 = 6$ m, there will be a 24% decrease in $(L/D)_h$, then $X_{24} = 0.0125$; and if the situation is 20% less severe for $\beta_T + \mu = 180°$, then $X_{25} = 1.0$ and $X_{26} = 0.2$.

During initial towing tests (Fig. 8) over the range of $V \cong 2$ to 5 m/sec (7–15 fps), a 50-lb drag force component existed. There was about a 30-lb athwartship force and a 20-lb downward force due to the tow rope inclination (a spring scale measured a total maximum force over the speed range of about 65 lb). Using the fact that the craft itself, in its tow-test configuration, weighed about 530 lb, it was determined that $\cos\gamma = 0.06$. There was no appreciable degradation of $(L/D)_h$ due to the height of the seas, which were on the order of a foot or so. Thus, the computed $(L/D)_h$ is given at about $V = 5$ m/sec;

$$(L/D)_h = (17.0 - 2.19 - 0.0715)[1 - (0.2)(0.06)] = 14.6$$

as opposed to a measured value of $530/50 \approx 10$. As mentioned in the text, this discrepancy is attributed, at least in part, to an inadequate towing rig. The situation will be rectified in future tests so that the forward float will not be pulled down into the water. Again, it is recognized that in the initial tow tests, the hydrofoils were set at too large an angle of attack, certainly not at an angle that would serve to maximize the lift-to-drag ratio. Because of the pivot design, this problem also can be easily eliminated.

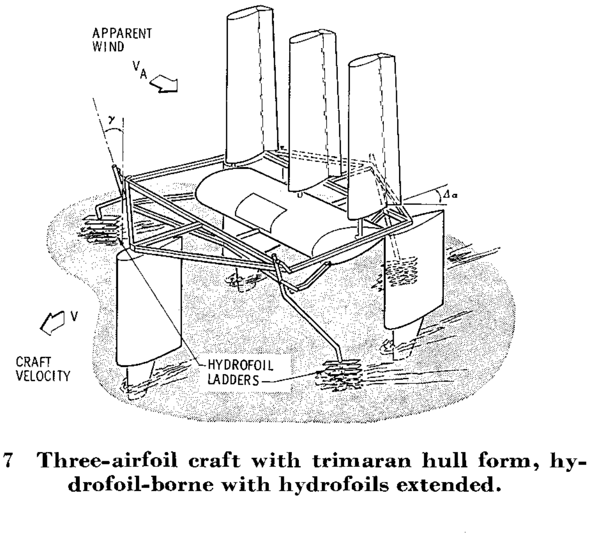

Fig. 7 **Three-airfoil craft with trimaran hull form, hydrofoil-borne with hydrofoils extended.**

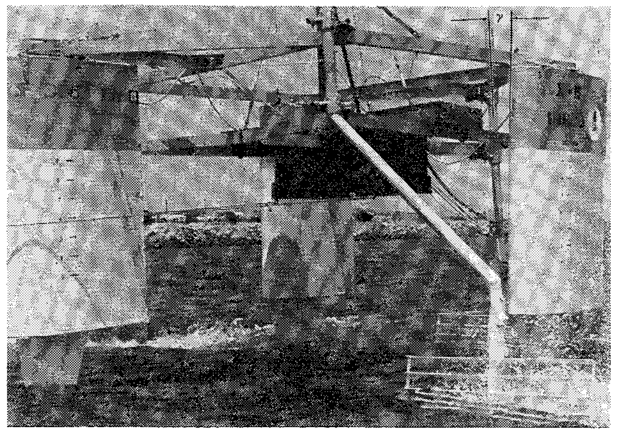

Fig. 8 **Bow view of prototype craft exhibiting the strut angle, γ.**

The hydrodynamic drag of a submerged hull is also rather complicated, and empirical data from large towing basins for particular hulls give rise to the only truly valid data.[17] Theoretically, the hull drag D_H is given by

$$D_H = \tfrac{1}{2}\rho_w A_H C_D V^2 \qquad (7)$$

where ρ_w is the water density (1,030 kg/m³ for sea water), A_H is the effective area of the hull for drag (m^2), and C_D is the overall hydrodynamic, hull-drag coefficient.

The drag coefficient is basically composed of a skin-friction component (function of Reynolds number, Re) and a wave-drag component (function of Froude number, Fr). The skin-friction component, C_{DS} tends to decrease logarithmically with Reynolds number,[18] which in turn is proportional to V. Specifically $Re = \rho_w V L \mu_w$, where L is the characteristic craft length and μ_w is the viscosity. Thus, one can make the approximation that

$$C_{DS} = \{1 - \exp[-(C_{DS})_1/Re]\}[(C_{DS})_0 - (C_{DS})_\infty] + (C_{DS})_\infty \qquad (8)$$

where $(C_{DS})_0$ = friction drag at $Re = 0$ and $(C_{DS})_\infty$ = friction drag at $Re \to \infty$.

The wave-drag coefficient, C_{DW}, is zero for $V = 0$ (no energy going into the waves) and increases approximately logarithmically with the Froude number, (Fr),[19,20] which also is proportional to V; specifically, $Fr = V/(gL)^{1/2}$. Thus one can make the series approximation

$$C_{DW} = (C_{DW})_0\{\exp[(C_{DW})_1 Fr] - 1\}\ \ldots \qquad (9)$$

In actuality, the wave-drag coefficient does not increase monotonically, but rather exhibits an undulating amplitude as the speed increases, with peaks appearing at various wave resonant frequencies. Because of the trimaran design stud-

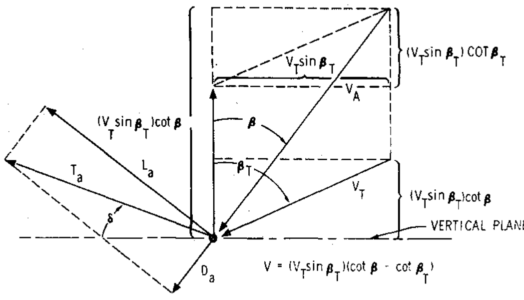

Fig. 9 Trigonometric relationships.

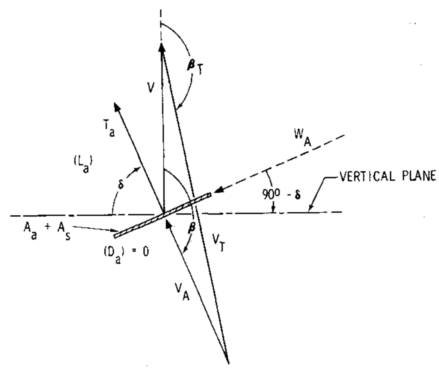

Fig. 10 Vector diagram of flat-plate configuration.

ied here, a destructive interference among the wave patterns is produced by the three hulls that at low speeds (prior to the craft reaching optimum lift-off speed, about 2.74 m/sec or 6 mph; (see Ref. 8, p. 1274) can be interpreted as the craft exhibiting a larger effective length, L. Thus Eq. (9) seems valid in the prelift-off speed regime that is relevant to this analysis if one arbitrarily increases L for the calculation of Fr.

In summary, a first approximation to hull-borne drag is

$$D_h \cong \tfrac{1}{2}\rho_w A_H\Big((1 - \exp[-(C_{DS})_1/Re])[(C_{DS})_0 - (C_{DS})_\infty] + (C_{DS})_\infty + (C_{DW})_0\{\exp[(C_{DW})_1 Fr] - 1\}\Big)V^2 \quad (10)$$

The aerodynamic lift-to-drag ratio of the craft, $(L/D)_a$, is also defined by a number of variables such as the drag of the superstructure of the craft, adjacency effects for multiple parallel airfoils, use of airfoil high-lift devices, variation of the wind above the water surface,[17,20,21] etc. (The vertical wind variation can be accounted for by a span-wise twist or angle-of-attack variation along the airfoil span. See $\Delta\alpha$ in Fig. 7.) In its simplest form one can define the lift-to-drag ratio by

$$(L/D)_a = A_a C_{La}/(A_s C_{Ds} + A_a D_{Da}) \quad (11)$$

where A_a is the area of the airfoil(s) in m^2, C_{La} is the lift coefficient of the airfoils, A_s is the area of the superstructure (including cabin, structural beams, etc.) in m^2, C_{Ds} is the mean drag coefficient of the superstructure, and C_{Da} is the drag coefficient of the airfoil(s) including skin, friction and induced drag.

Although C_{Ds} is variable and depends upon Reynolds number and the aspect that the craft presents to the apparent wind, the overriding variation, given C_{La}, is the induced drag portion of C_{Da}.

The basic, classical equation[22] is

$$C_{Da} = (C_{Da})_0 + C^2_{La}/\pi AR(1 + \sigma) \quad (12)$$

where $(C_{Da})_0$ is the profile drag coefficient of the airfoils, e.g. Eq. (28) [and varies as a function of Reynolds number and lift-coefficient—including especially the influence of slots and flaps, etc.],[23] AR is the "effective" aspect ratio of the airfoils,§ and σ is a factor that accounts for the nonellipticity of the airfoil's planform (e.g., for rectangular airfoils, $\sigma = 0.054$).[25]

Having developed analytical approximations to all of the pertinent parameters affecting the hydrofoil/airfoil craft's performance, we can now set about to determine the equations of motion. In the case of conventional, hull-borne sailcraft, the balance between the forward component of aerodynamic thrust and the aft component of hull drag is given by Eq. (13), where T_a is the total aerodynamic thrust (in Newtons) and δ is the angle between the aerodynamic thrust vector, $\mathbf{T}_a$, and a vertical plane normal to the craft's velocity vector, $\mathbf{V}$, (Fig. 6). That is

$$D_H = T_a \sin\delta \quad (13)$$

The thrust is given by

$$T_a = +(L_a{}^2 + D_a{}^2)^{1/2} = \tfrac{1}{2}\rho_a A_a V_A{}^2 C_{La}(1 + 1/(L/D)_a{}^2)^{1/2} \quad (14)$$

where L_a is the lift of the airfoil(s) in Newtons, D_a is the total drag of the craft [superstructure and airfoil(s)] in Newtons, ρ_a is the air density (normally about 1.22 kg/m³), and V_A is the apparent wind speed in m/sec.

From the law of cosines the apparent wind speed is given by

$$V_A{}^2 = V^2 + V_T{}^2 - 2VV_T\cos(180° - \beta_T)$$

or

$$V_A{}^2 = V_T{}^2(1 + S^2 + 2S\cos\beta_T) \quad (15)$$

where V_A is the apparent wind speed in m/sec, $S \triangleq V/V_T$, V_T is the true wind speed in m/sec, and β_T is the angle between the direction from which the true wind is coming and the velocity of the craft, $\mathbf{V}$.

We next introduce the angle, β, (measured between the direction from which the apparent wind is coming and the velocity of the craft) and relate it to S and β_T. From trigonometry, as applied to Fig. 9, $V_T\sin\beta_T = V/(\cot\beta - \cot\beta_T)$ so that

$$\tan\beta = \sin\beta_T/(\cos\beta_T + S) \quad (16)$$

From Fig. 9

$$\cot^{-1}(L/D)_a + \delta + 90° - \beta = 90°$$

or

$$\delta = \beta - \cot^{-1}(L/D)_a \quad (17)$$

where δ must lie in the range $0 < \delta < 180°$ in order for propulsion to be possible. Elimination of β between Eqs. (16) and (17) yields

$$\delta = \tan^{-1}\{\sin\beta_T/(S + \cos\beta_T)\} - \cot^{-1}(L/D)_a \quad (18)$$

From signs of $\sin\beta_T$ and $S + \cos\beta_T$ one can determine proper quadrant for $\tan^{-1}$.

Having developed the appropriate geometrical relationships, the next order of business will be to develop another equation for the aerodynamic thrust, T_a, required to overcome the hydrodynamic drag. If L_h is the lift component of the hydrofoil force vector, then for equilibrium in the vertical plane (Fig. 5)

$$L_h = +(T_a{}^2\cos^2\delta + (mg)^2)^{1/2} \quad (19)$$

Similarly, equilibrium in the horizontal plane is governed by Eqs. (7) and (13). The division of Eq. (19) by Eq. (13)

§ From p. 221 of Prandtl,[24] the aspect ratio of two adjacent airfoils is enhanced by the factor $1/k$, in which $1/k = [1 + 3.5(h/s)]/[0.975 + 1.44(h/s)]$, where h is the distance between airfoils and s is the airfoil span. This relationship is only useful in the region $s/2 > h > S/15$ and is assumed to be a conservative approximation for the vertical triplane configuration studied here.

yields

$$(L/D)_h = L_h/D_h = [T_a^2 \cos^2\delta + (Mg)^2]^{1/2}/T_a \sin\delta]$$

or

$$T_a = Mg/[(L/D)_h^2 \sin^2\delta - \cos^2\delta]^{1/2} \quad (20)$$

It is obvious from Eq. (20) that in order to avoid a negative radicand

$$\sin^2\delta(L/D)_h^2 > \cos^2\delta \quad (21)$$

Such a mathematical conclusion is directly related to the physical fact that a sailcraft cannot "point" too close to the wind. If it does head too close to the wind then there is no component of $\mathbf{T}_a$ in the forward direction sufficient to provide propulsive force [i.e., δ becomes so small that Eq. (13) isn't satisfied]. Clearly, if the inequality, Eq. (21), does not hold, then the calculation is meaningless and should not be continued. The basic equation governing the steady-state motion of the hydrofoil-suspended sailcraft using rigid airfoil(s) for lift is obtained by a combination of Eqs. (14), (15), and (20):

$$\tfrac{1}{2}\rho_a A_a C_{La} V_T^2(1 + S^2 + 2S \cos\beta_T) \times \{[1 + 1/(L/D)_a^2][\sin^2\delta(L/D)_h^2 - \cos^2\delta]\}^{1/2} = Mg \quad (22)$$

in which δ is obtained from Eq. (18) and the lift-to-drag ratios from Eqs. (2), (11), or (12). If the radicand becomes negative, then this transcendental equation cannot be solved since the craft is pointing too high into the wind. Otherwise, Eq. (22) can be solved for $(S > 0)$, given A_a, C_{La}, V_T, β_T, and Mg.

Figure 10 indicates the general vector diagram for the airfoil(s) employed as a flat plate always oriented normally to the apparent wind (similar to a conventional sailboat moving on a run with the wind aft). The equation governing the aerodynamic thrust in the aforementioned situation is

$$T_a = \rho_a[(A_a C_{Dap} + A_s D_{Ds})/2]V_a^2 \quad (23)$$

where C_{Dap} is the drag coefficient of the airfoil(s) with chords normal to the apparent wind (acting as a flat plate).

Since the airfoil planform(s) are faced normal to the wind, one can shift the wind vector diagram 90°, establish a fictitious apparent wind, W_A, and substitute 90°-δ for δ, T_a for L_a, and zero for D_a (Fig. 10). Thus, the basic equation governing the steady-state motion of the hydrofoil-suspended sailcraft using the rigid airfoil(s) as a flat plate (on a run for example) is obtained by a combination of Eqs. (15), (20), and (23), with

$$(L/D)_a \to \infty,\ \delta = \beta\ [\text{Eq. (17)}],\ \sin^2\delta \to \cos^2\beta, \text{ and } \cos^2\delta \to \sin^2\beta \quad (24)$$

so that

$$\rho_a\{(A_a C_{Dap} + A_s C_{Ds})/2\} V_T^2(1 + S^2 + 2S \cos\beta_T) \times (\cos^2\beta(L/D)_h^2 - \sin^2\beta)^{1/2} = \mathrm{Mg} \quad (25)$$

in which β is obtained from Eq. (16) and $(L/D)_h$ from Eq. (2).

Similar equations can be developed for nonhydrofoil-borne operations, which are required for sailboat speeds below optimum liftoff speed, $V^*_{\ell_0}$ (see Baker[8]). In this case D_h is given by Eq. (10) and must be equated to Eq. (13); in addition, Eqs. (14) and (15) must be introduced (in this case we'll call the craft speed, V_1)

$$\tfrac{1}{2}\rho_w A_H\{[(C_{DS})_0 - (C_{DS})_\infty]\{1 - \exp[(C_{DS})_1/Re]\} + (C_{DS})_\infty + (C_{DW})_0 (\exp[(C_{DW})_1 Fr] - 1)\} V_1^2 = \tfrac{1}{2}\rho_a A_a C_{La} V_T^2(1 + S^2 + 2S \cos\beta_T) \times (1 + 1/(L/D)_a^2)^{1/2} \sin\delta \quad (26)$$

($S \triangleq V_1/V_T$ in this case). Equation (26) is valid for the steady-state motion of a conventional sailcraft using rigid airfoils for lift. The analysis for the use of rigid airfoils as a flat plate (run configuration) is

$$\tfrac{1}{2}\rho_w A_H([(C_{DS})_0 - (C_{DS})_\infty]\{1 - \exp[(C_{DS})_1/Re]\} + (C_{DS})_\infty + (C_{DW})_0 (\exp[(C_{DW})_1 Fr] - 1)) V_1^2 = \tfrac{1}{2}\rho_a(A_a C_{Dap} + A_S C_{Ds}) V_T^2(1 + S^2 + 2S \cos\beta_T) \cos\beta \quad (27)$$

Equations (25) and (27) can be solved for a positive S provided that $\beta > 90°$ (winds abaft of the beam) and the radicand of Eq. (25) is positive.

Discussion of Analysis

Prior to exercising the algorithm, it is important to understand the logic by which the four alternative sets of equations of motion [Eqs. (22), (25), (26), and (27)] are employed. Our first concern is with the lift off speed V_{ℓ_0}, i.e., with the speed at which the craft is able to become hydrofoil borne. As has been established,[8] for any given craft there exists a unique, optimum lift off speed, $V^*_{\ell_0}$. Prior to the craft attaining this speed there is usually no advantage to be gained by immersing the hydrofoils, and conventional hull-borne operation (without submerged hydrofoils) is superior. Once having reached $V^*_{\ell_0}$, however, the hydrofoils should be immersed and hydrofoil-borne operation becomes superior to hull-borne operation for all higher craft speeds. At the higher end of the speed range, cavitation constraints come into play and, essentially, set an upper limit to craft speed. Once the cavitation speed constraint has been reached, the hydrodynamic lift-to-drag ratio must be arbitrarily decreased in order to keep the craft speed down and the hydrofoils cavitation free. Physically speaking, one would achieve this by reducing the hydrofoil's lift coefficient. Cavitation arises (for subcavitating hydrofoils) when the hydrofoils move at high speed and with relatively large loading. These circumstances trigger cavitation which in turn erodes the hydrofoil surface. Cannon[5] indicates that above 50 fps (about 15 m/sec) "...cavitation is probably present for loadings of over 600 psf..." On the other hand Wadlin et al.[26] in Figs. 5a and 5b indicate cavitation-free speeds up to 45 fps (14 m/sec) for $AR = 10$ hydrofoils at loadings of 1800–2000 psf. On page 4 of Ellsworth[13] it is noted that 2000 psf is the maximum loading and that if cavitation is really to be avoided the loadings should be in the neighborhood of 1200–1400 psf. As far as speed is concerned, Ellsworth states: "... it appears that speed much above 40–45 knots (20–23 m/sec) will always be associated with some cavitation. . ." Hence one turns to base-ventilated hydrofoils, supercavitating hydrofoils, etc. Ramseu[27] studied $AR = 0.25$ flat plates of different scale and thickness ratio as well as elliptic planform NACA 16-509 ski-type hydrofoils. For lift coefficients of about 0.1 he found the onset of cavitation between 18 and 24 m/sec. Myers[28] in Fig. 6 (for Boeing's "Little Squirt") indicates the inception of cavitation at 43 knots (22 m/sec) for a 16-206 hydrofoil with a lift coefficient of about 0.25. Figure 1 of Altmann[14] presents subcavitating hydrofoil data up to 80 knot speeds (41 m/sec), but it is not at all clear that cavitation is not present at such high speeds. On page 40 Wadlin et al.[29] states: "At a speed of 30 fps (9 m/sec) no cavitation has occurred . . . 40 fps (12 m/sec) indicates the beginning of cavitation . . . The area of pressure relief may have existed from the start of the cavitation but was so small as to be unnoticeable before a speed of 62 fps (19 m/sec) was reached." In fine, we will set the critical maximum speed for cavitation-free motion, V_c, at 18 m/sec (60 fps). For higher speeds one would naturally not wish to employ subcavitating hydrofoils, but, on the other hand, a small sailcraft would probably not often encounter winds high enough to accelerate it beyond 18 m/sec (40 mph), and more sophisticated hydrofoil systems are useful only for larger craft in which case V_c is set at 26 m/sec (85 ft/sec). The influence of water roughness on V_c must also be accounted for by an empirical calculation.

Of course structural constraints will come into the picture, and they could be reflected by imposing a limiting aerodynamic thrust, $T_{a\,\max}$. As can be seen by Eq. (14), T_a can be reduced by reducing $(L/D)_a$ down to, at most, zero by reducing C_{La} [Eq. (11)]. Note that $(C_{Da})_0$ also changes[23] approximately in accordance with

$$(C_{Da})_0 = \{0.0353/[12.68 - \exp(C_{La})] - 0.0222\} \quad (28)$$

(On a run the constraint of T_a is usually less, since one is running before the wind.) After reaching $T_{a\,\max}$ one is in a storm condition and the craft cannot operate in an optimal fashion because safety becomes an overriding concern. Modeling safety constraints lies outside the scope of the present analysis and will not be pursued. In summary: 1) if solution of Eq. (22) gives rise to a $V(=V_TS)$ less than $V^*_{\ell 0}$, then one turns to Eq. (26); 2) if a solution of Eq. (22) gives rise to a V that exceeds the cavitation constraint, then $(L/D)_h$ is reduced until V is sufficiently reduced; and 3) if Eq. (14) gives rise to an unacceptably large aerodynamic force, T_a, then $(L/D)_a$ is reduced until either T_a is reduced to the limiting size or $(L/D)_a = 0$, at which point the craft is assumed to be inoperative and V is set to zero.

When the apparent wind is abaft the beam, better performance can sometimes be achieved with the airfoil(s) in the flat plate configuration. Thus for $\beta > 90°$, Eq. (25) should be solved in parallel with Eq. (22) and the solution yielding the larger S should be adopted. If the V obtained by the larger S exceeds cavitation constraints, then $(L/D)_h$ should be reduced. If T_a is too large, then A_a should be reduced until Eq. (23) shows that T_a is within limits. Safety concerns when "running before the wind" are ordinarily less severe and the craft may be assumed to be operative until A_a is forced to zero and inoperative thereafter. If even the larger of the V's obtained by Eqs. (25) or (22) is below $V^*_{\ell 0}$, then Eq. (26) should be employed (alone, until $\beta > 90°$ and in parallel with Eq. (27) thereafter). When the craft points too high into the apparent wind, it is also inoperative. This effect evidences itself by the radicands of either Eq. (22), (25), or (26) becoming negative. Under such a circumstance, the craft cannot make headway and V is set to zero. If $S \leq 0$, the craft also makes no headway.

Special Cases and Conclusions

In order to gain insight into the optimization problem, it is useful to render Eq. (22) into a simpler form in which we specialize to a beam reach with $\beta_T = 90°$. For large effective-aspect-ratio rigid airfoils we are usually justified in setting

$$\{[1 + 1/(L/D)_a{}^2]\}^{1/2} \cong 1 \quad (29)$$

and assuming that $1/(L/D)_a \ll \beta$. Thus Eq. (22) reduces to

$$(\rho_a A_a C_{La}/2)V_T{}^2(1 + S^2)(\sin^2\delta(L/D)_h{}^2 - \cos^2\delta)^{1/2} = \mathrm{Mg} \quad (30)$$

where $\delta \cong \beta \cong \tan^{-1}(1/S)$.

Considering the case of high performance, i.e., $S > 1$, but with an S not so large as to violate the assumptions that $1/(L/D)_a{}^2 \ll 1/S$ and $(L/D)_h{}^2(1/S^2) \gg 1$, we have the approximation that

$$(\rho_a A_a C_{La}/2)V_T{}^2 S(L/D)_h = \mathrm{Mg} \quad (31)$$

or, since $S = V/V_T$, the required true wind speed V_T is

$$V_T = 2\mathrm{Mg}/\rho_a A_a C_{La}(L/D)_h V \quad (32)$$

The three following conclusions can be drawn from Eq. (32): 1) equation (32) exhibits the paradox that the ability to operate in light winds on a beam reach is improved by large craft speed; 2) the intuitively evident advantage of small weight-to-airfoil(s)-area quotients, Mg/A_a, is placed in quantitative terms; and 3) on a beam reach it is apparent that a large C_{La} is relatively more important than a large $(L/D)_a$ since the $(L/D)_a$ term is out of Eq. (32). Thus, one can find advantage in symmetrical high-lift wing devices on a beam reach even if they degrade the lift-to-drag ratio.

When pointing high into the wind (small β), a large $(L/D)_a$ is required in order for the craft to make headway. One can establish the minimum permissible β by considering that for small β and β_T, Equation (16) reduces to

$$\beta \cong \beta_T/(1 + S)$$

so that

$$(\beta_T)_{\min} = \beta_{\min} + S\beta_{\min} \quad (33)$$

From Eq. (17) for large $(L/D)_a$, we find

$$\delta \cong \beta - 1/(L/D)_a$$

so that

$$\beta_{\min} = \delta_{\min} + 1/(L/D)_a \quad (34)$$

Again assuming large $(L/D)_a$, a positive radicand in Eq. (22) requires that $\tan^2\delta > 1/(L/D)_h{}^2$ so that

$$\delta_{\min} > 1/(L/D)_h \quad (35)$$

A combination of Eqs. (34) and (35) gives rise to a specification of the minimum permissible β, that is

$$\beta_{\min} > 1/(L/D)_h + 1/(L/D)_a \quad (36)$$

It can, therefore, be concluded that a large $(L/D)_a$ (as well as, of course, a large $(L/D)_h$) is vital in order to point high into the wind. [It should be noted that a maximum $(L/D)_a$ is achieved when $A_sC_{Ds}/A_a + (C_{Da})_0 = (C_{La}{}^2/\pi AR)(1 + \sigma)$.] Because of Eqs. (11) and (12), such a requirement is not compatible with a large C_{La} (because of induced drag); consequently, an optimization based upon a variation in C_{La} is required. It is a property of the vertical triplane and adjustable flap design of the rigid airfoils, plus the aerodynamically shaped floats, that one can allow for the required tradeoff variation between large C_{La} and $(L/D)_a$.

The aerodynamically shaped floats provide a 50% increase in airfoil area (dependent upon variation of the wind above the water surface[17,20,21] with the craft on a beam reach and hydrofoil suspended. This effect was confirmed during initial tow tests of a full-scale prototype (Fig. 1) in which, once hydrofoil borne, the craft tried to sail itself with only the floats available for aerodynamic thrust. As the craft velocity, V, increases, the angle-of-attack of the floats, β_{bc}, becomes more favorable and their aerodynamic efficiency increases, thereby adding to the total aerodynamic thrust, $\mathbf{T_a}$, of the craft.

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